Option method for estimating exposures of linear derivatives.

# Swaption method

**Introduction and definitions**

Swaption valuations are commonly used to estimate future expected positive exposures (EPE) and expected negative exposures (ENE) for (single currency) interest rate swaps (IRS) for the purpose of CVA/DVA estimations. This is a popular method for estimating exposures IRS as:

1. The method is consistent with traded/observable market interest rate term structure and interest rate volatility surface.
2. Swaption valuations are simple to determine and not computationally extensive (compared to simulation-based approaches)

**Key definitions:**

|  |  |
| --- | --- |
| Value | Definition |
|  | The present day / the “valuation date” |
|  | The effective date of the IRS and the start of the 1st interest accrual period of the IRS. IRS are commonly traded as ‘immediate start’ so , is usually equal to or prior to however, IRS can be forward starting – i.e. the effective date is 1 month or 1 year after the trade date. |
|  | The termination date of the IRS |
|  | The start on the 2nd interest coupon accrual period of the IRS |
|  | The start on the 3rd interest coupon accrual period of the IRS |
|  | The payment date of cashflow, t of the IRS |
|  | The value of the IRS at time t,  if is a random variable |
|  | The expected positive exposure of the IRS at time t;  This is identical to the value of a bought swaption on the IRS. |
|  | The expected negative exposure of the IRS at time t  This is identical to the negated value of a bought swaption on the reversed IRS (i.e. where the pay/receive perspective is swapped). |
| payer swaption | A swaption on a ‘pay fixed, receive float’ IRS. The buyer (seller) of a ‘payer swaption’ has the option to enter (obligation to enter) the defined ‘pay fixed, receive float’ IRS, on the swaption expiry date |
| receiver swaption | A swaption on a ‘receive fixed, pay float’ IRS. The buyer (seller) of a ‘receiver swaption’ has the option to enter (obligation to enter) the defined ‘receive fixed, pay float’ IRS, on the swaption expiry date |
| Vanilla swaption | A vanilla swaption is defined as an immediate start, European swaption. Market convention for swaption’s. |
| European swaption | A European swaption only has 1 expiry date. Market convention is for a swaption to be a European swaption. |
| American swaption | An American swaption can be exercised on any date, prior and including the expiry date. This definition is included for completeness, however, American swaptions are not used in this paper’s explanation or implementation of the swaption method. |
| Immediate start swaption | A swaption where the effective date of the IRS is a settlement delay (typically 1-2 business days) after the expiry date of the swaption. |
| Forward swaption | A swaption where the effective date of the IRS is more than a settlement delay (1-2 business days) after the expiry date of the swaption. For example, the expiry of the option could be 1 month or 1Y prior to the effective date of the IRS. |

**Geneal methodology**

For all future dates, , (i.e. after the present date and before the termination date ( of the interest rate swap), the exposure of the IRS can be estimated via a European swaption valuation, where the underlying swap of the swaption is defined by the outstanding coupon periods.

For the set of dates one settlement delay (typically 1-2 business days) prior to the next cashflow period of the IRS (i.e. , the exposure of the IRS, can be estimated with a European, immediate start swaption.

For all other dates, the exposure of the IRS, must be estimated with a European, forward start swaption.

For dates, (i.e. after the present date and before the effective date of the IRS), the defined IRS of the swaption, is identical to the IRS we are estimating the EPE / ENE for.

For dates (i.e any date after the 1st coupon period has completed), the underlying IRS of the swaption must exclude the historically settled coupon periods.

Please note this explanation has assumed a zero-settlement delay of coupon settlement after the coupon accrual period finishes. Specific consideration of such details is fiddly and is not the purpose of the brief.

**Implementation**

The EPE/ENE estimates for a given date, require two swaption valuations, one for the EPE and one for the ENE. For example, if the IRS has 2000 days until termination, if we wanted to estimate the EPE/ENE on each day, 4000 swaption valuations are required.

Swaption valuations are computationally cheap, though most practitioners will choose a “grid” of dates (i.e. weekly, monthly or some term structure of decreasing granularity). As the pillar instruments of the applicable market data inputs (interest rate curves and interest rate volatility surfaces) have a limited granularity, increasing a date grid granularity beyond (i) the granularity of the market data and (ii) the cashflow settlement frequency of the IRS has is unlikely materially impact results.

**References**

[1] Swaption Approach <https://web.archive.org/web/20230603155222/https://sp-finance.e-monsite.com/pages/risk-management/cva/credit-risk-modeling-for-derivatives/swaption-approach/swaption-approach.html>

# FX option method

The FX option method is inspired by the “Swaption” method and is applicable for estimating the value of linear FX derivatives such as FX forwards. Like the swaption method, the benefits are:

1. The method is consistent with traded/observable market FX term structure and the FX volatility surface.
2. FX option valuations are simple to determine and not computationally extensive (compared to simulation-based approaches)

In the swaption method, we can take the swaption valuations, as a direct estimation of the IRS EPE/ENE at a given date. However, in the FX option method, this not appropriate.

**Key definitions:**

|  |  |
| --- | --- |
| Value | Definition |
|  | The present day / the “valuation date” |
|  | The contractual rate / strike (perspective Base/Quote, i.e. # of quote currency units per 1 base currency unit) |
|  | The maturity date of the FX forward |
|  | The market discount factor between time and |
|  | The market FX forward rate at time T (perspective Base/Quote, i.e. # of quote currency units per 1 base currency unit) |
|  | Signed notional amount in the base currency (has opposite sign to |
|  | Signed notional amount in the quote currency (has opposite sign to |
|  | The value of the FX forward (with maturity and strike K) at time  if is a random variable  where the result is in the quote currency |
|  | The value of an FX option (with defined maturity and strike K) at time t, where the result is in the quote currency:  (assume discount rates are deterministic so can move outside the expectation)  (add constant K to inside the max function, subtract it after the expectation)  (divide by constant K inside the max function, multiply it back after the expectation)  **[Equation 1]**  (move the constant K to the other side of the integral) |
| EPE(t) | The expected positive exposure of the FX forward at time t:    (substitute in the definition for from above)  (rearrange as is a constant and as assumed to be deterministic)  (add 1 to inside the max function and subtract 1 outside the max function)  (move 1 to the other side of the integral)  **[Equation 2]** (move K through the brackets) |
| ENE(t) | The expected negative exposure of the FX forward at time t |

Observe the identical integral terms in equations 1 & 2 the table above.

This means from an FX option valuation with

* the strike per the FX forward contractual rate,
* the same currency pair perspective as the FX forward
* setting the expiry date of the option per the expiry date (also known as the rate fixing date) of the FX forward.

is a perfect model of the EPE as at the maturity date of the FX forward.

For the ENE scenario, we simply value a sold FX option with a reversed call/put perspective (compared to the EPE scenario).

**Methodology for estimating the EPE at any date, t**

The following describes the methodology for determining expected positive exposure of FX forward at date t (which can be any date, after the value date, and equal to or prior to the FX forward expiry date).

Value a bought, same call/put currency perspective vanilla European FX option with

* the strike per the FX forward contractual rate adjusted by the forward point difference between the FX forward expiry date and date t,
* the same currency pair perspective as the FX forward
* setting the expiry date of the option, to date

The value of this FX option (in the quote currency) is:

If the option valuation is nil, there is no EPE scenario, so EPE is nil. Otherwise, if the valuation is not nil, we can extract the effective FX rate , through a rearrange of the equation:

This rate is then adjusted by the FX forward point delta between the grid date t and the maturity date, , to adjust it to the FX forward rate for the maturity date of the FX forward instrument.

This FX forward rate can then be used in the standard FX forward valuation formulae to get the expected positive exposure at date t.

This methodology can be applied across a grid of dates, from just 1 day after the present day / valuation date, all the way to the maturity date of our FX forward.

**Methodology for estimating the ENE at date, t**

The methodology for estimating the ENE, is similar. The sole change is the call/put currency perspective is reversed.

**Methodology issues with this FX option method**

This method defined above seems generally intuitive and aligns to practical checks with FXF simulations via GBM / Heston.

However, we do not have a mathematical proof that this is correct.